

## Presenting an Energy Efficiency Model in the Iranian Economy Using Dynamic Optimization Approach

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### Abstract

This paper is based on dynamic optimization methodology to investigate the economic energy efficiency issues in Iranian Economy. The paper introduces some definitions about energy efficiency both in economics and physics, and establishes a quantitative way for measuring the economic energy efficiency.

The linkage between economic energy efficiency, energy consumption and capital stock is demonstrated primarily. Using the methodology of dynamic optimization, a maximum problem of economic energy efficiency over time, which is subjected to the extended Solow growth model and instantaneous investment rate, is modelled. In this model, the energy consumption is set as a control variable and the capital is regarded as a state variable. The analytic solutions can be derived and the diagrammatic analysis provides saddle-point equilibrium.

**Keywords:** Economic energy efficiency, optimum consumption of energy, Iran's economy.

**Jel Classification:** C14, C61, J24, O53.

### 1. Introduction

International concern over environmental issues such as global warming and climate change has put severe political and economic pressures on governments of both developed and less developed countries. Energy efficiency is one of the relevant targets to be met by international environmental standards. Although a large number of studies have discussed the optimal path to economic growth, energy consumption or pollution reduction, economists have rarely presented their views on energy efficiency issues. In fact, improving energy efficiency is a dynamic procedure in development and has been always related to growth, investment, technology change and many other economic variables.

Energy efficiency can be defined in three dimensions. The first definition stems from the thermodynamic laws in physics. The second definition is based on economic concepts and named energy intensity, which is the ratio of energy

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input to aggregate output (Giacone and Manco, 2012). However, this definition considers energy as a unique input ignoring the other factors in its production such as capital and labour. Stern (2012) has developed this definition of economic energy efficiency under the multi-input framework. Fig. 1 demonstrates a system consisting of economic growth, energy consumption, energy efficiency and capital.

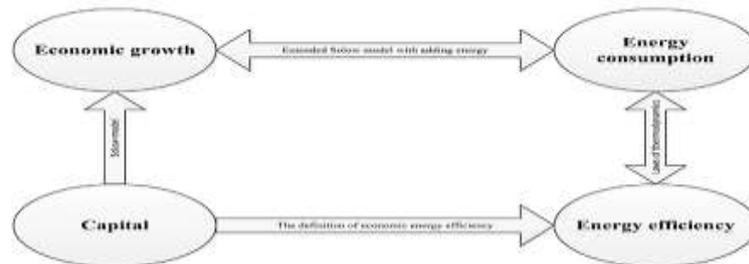


Fig. 1. The mechanism and linkage between energy efficiency and other economic variables.

## 2. Background

Dynamic optimization has been applied by Pindyck for resource exploitation problems since 1970s. He established a basic model on the optimal exploration of non-renewable resources (Pindyck, 1978) and developed it in 1980 by adding uncertainty into exhaustible resource market analysis (Pindyck, 1980). Both models are based on cost benefit analysis. Basically, they are general models and focus only on the optimization in production. The limitation associated with them is that the energy depletion has not been linked with economic growth and any other macroeconomic variable. Some other economists including Stiglitz (1974), Garg and Sweeney (1978), Dasgupta and Heal (1980) brought the optimal exploration problem into the framework of neoclassical model of growth. They discussed the optimal sustainable growth path under the condition that the resources are scarce and diminishing all the time. But the technology element was assumed exogenous in their models, which has aroused a wide controversy. After the endogenous growth model was raised by Romer (1988) and Lucas (1988), the technology change became endogenous so that the long-run analysis was feasible and reasonable for further analysis. Nevertheless, the literature on endogenous growth model rarely includes the natural resources problems. To some extent, they assume that natural resources are inexhaustible in supply and are always available for use. While in the last two papers on China's issues by Peng (2007) and Li (2012), the authors developed the endogenous growth model treating natural resources as a constraint and got an optimal path of development eventually. However, they do not mention the issues about energy efficiency.

### 3. Methodology

Optimized energy efficiency is hard to discover mainly because economic energy efficiency is hard to be quantified appropriately. In this section, the modeling will start with a quantitative definition given by mathematical and geometrical method. Then, Solow model will be extended taking energy consumption into account. After these preparations, the dynamic maximum problem with some constraints will be modeled. As illustrated in Fig. 1, economic energy efficiency is linked with energy itself and capital. Thus, economic energy efficiency can be defined as a function of capital stock and energy consumption under the multi-input framework. Denote economic energy efficiency by  $W$ , energy consumption by  $E$  and capital stock level by  $K$ . The basic mathematical expression of economic energy efficiency is:

$$W = W(E, K)$$

Obviously, economic energy efficiency depends on energy use. For any given level of output under any level of technology, the more energy we use, the lower economic energy efficiency will be. The reasons that we included capital in the model can be explained from the stock and flow angles perspectives. For one thing, the level of capital stock determines the level of development and technology, which is the basis of economic energy efficiency improvement. For another, the level of capital stock is directly associated with investment, which is the key driver of economic growth and technology progress in developing countries. Thus, capital is tightly related to economic energy efficiency. In this model, technology is considered as an exogenous variable embodied by investment and capital stock level. With energy and capital as the two inputs, to some extent, economic energy efficiency is similar to the production function. On the other hand, certain levels of economic energy efficiency can be evaluated and compared, as in the methods used in the theory of ordinal utility. This definition is demonstrated geometrically in Fig. 2. In Fig. 2, the three curves represent three levels of economic energy efficiency with different input combinations of capital and energy under the same output level. All points on the curves are efficient states. The direction of the arrow indicates the direction of increase in economic energy efficiency, implying 'the less, the better' for inputs. Given the level of output under technology confine, the less inputs use means the more efficient the energy economic system is.  $W$  stands for economic energy efficiency, thus,  $W_3 > W_2 > W_1$ . The curves in Fig. 2 are concave rather than convex mainly because of the diminishing marginal rate of substitution. It shows how much capital input increase can substitute one unit of energy we reduce. It is similar to the production possibility curve.

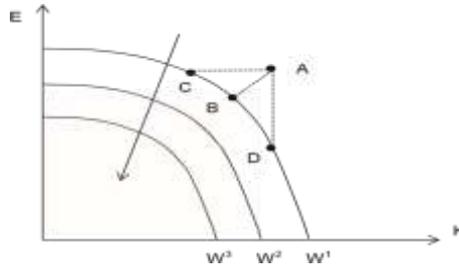


Fig. 2: The economic energy efficiency

### 3.1. Optimization of energy efficiency

In a developing economy, all levels of energy efficiency can be maximized over time, assuming that different levels of energy efficiency increase over time. Maximum energy efficiency model with growth and energy constraints is presented below:

$$\begin{aligned} & \text{Max} \int_0^T W(K, E) dt \\ & \text{s. t. } Y = (1 - \gamma)K^\beta L^{1-\beta} + \gamma E \\ & \dot{K} = s(Y - E) - \delta K \\ & K(0) = K_0 \quad K(T) \text{ Free} \end{aligned}$$

### 4. Conclusion

The modeling on the dynamic optimization of economic energy efficiency carries some implications. In development, the economic energy efficiency is a factor which is correlated with other macroeconomic variables. On one hand, industrialization and modernization lead to the appetite for energy. The improvement of economic energy efficiency is essential for effective use and conservation of energy. On the other hand, the improvement of economic energy efficiency depends largely upon the capital stock, investment, and technology, which are the key drivers of development. Pursuing the maximum economic energy efficiency is not contradictory with investment and energy consumption. In contrast, they can be harmonized in economic growth and development. This can help the developing countries to achieve a sustainable development.

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