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Calculation of the Steady-State Production Trajectory for Iranian Economy (An Endogenous Growth Model With CES Production Function)

Ostadzad, A. H.[1*]

Abstract

The Cobb Douglas production function has been used in most economic growth models as production technology, which has elasticity of substitution equal to one. In this study, the Ramsy growth model is extended with CES production function (production function with constant elasticity of substitution, not one). After solving the optimal control problem, the optimal trajectory of economic variables (production and capital) was calculated in steady state for the Iranian economy. The results showed that production and capital are significantly lower than the steady state level. One of the innovations of this paper is the investigation of the effect of elasticity of substitution on optimal trajectory of economic variables as well as economic growth in steady state. The results indicated that the elasticity of substitution will have a negative impact on the growth in the steady state. Theoretical results of the study show that assuming elasticity of substitution equal to one, the technology growth rate does not have a significant effect on the economic growth rate in the long run. But given elasticity of substitution less than one, the steady growth rate of production will depend on technology and population growth rates. Therefore, considering the CES production function rather than the Cob Douglas in the growth model would yield quite different results. Also in this research the economic growth rate was calculated to be 6.9 percent for the Iranian economy. The results further showed that the production realized for Iran had not been in a steady state trajectory and Iranian economy was in deep recession in 2018. The reason for this recession was reduced investment.

Keywords: CES Production Function, Growth models, Genetic Algorithm, Constant Elasticity of Substitution

JEL Classification: O41, O40, D24, B55

1. Introduction

In the present study, the long-term economic growth rate and also the optimal trajectory of macroeconomic variables (production, capital) were investigated using the Ramsey growth model assuming a Ramsy growth model with CES production function (constant elasticity of substitution production function). In most of the existing studies on the growth, we have the assumption of the production function with elasticity of substitution equal to one (Cobb Douglas

1. Assistant Professor and Faculty of University of Larestan, Lar, Iran **Email:** a.ostadzad@lar.ac.ir

production function). This assumption is limiting for the elasticity of substitution and the elasticity of substitution of production factors for each country cannot be equal one in the long run.

Christensen (2011) developed an exogenous growth model with an increasing returns-to-scale production function. In Christensen's model, it has been shown that per capita output growth will be positive even if the population growth rate is negative. In this study, it has been also shown if we have a Cobb Douglas production function assuming a constant returns-to-scale, the long-run per capita growth rate will be positive even if the technology's growth rate is assumed to be zero. These results indicate that the technology improvement may not be important in the long-term economic growth. Christensen's research results could be due to the Cobb Douglas production function in the extended model. The results of the present study also confirmed the results of Christensen (2011) by assuming a Cobb Douglas production function as a specific case. But when the production function is not Cobb Douglas and assuming a constant elasticity of substitution (CES production function), the engine of long-term economic growth is growth rate of technology.

Several empirical studies have been suggested less-than-one values for elasticity of substitution (Chirinko 2008). Based on these experimental findings, in this study, as against the previous studies in the field of growth, the production function was assumed to be a constant elasticity of substitution (CES production function) instead of the Cobb Douglas. The research results show that by assuming elasticity of substitution equal to one, the growth rate in steady state will be equal to the population growth rate, and the technology growth is not important in the long run, and it will not affect steady state economic growth. But when the elasticity of substitution is less than one, the economic growth in the steady state will depend on the technology and population growth rate. So, we will have quite different results when considering the CES production function instead of the Cobb Douglas in an exogenous growth model. Therefore, this study has answered the questions of "what is the elasticity of substitute factors for the Iranian economy? and "does the considering elasticity of substitution as less than one have a significant effect on economic growth in the steady state?

2. Theoretical Background and Literature Review

In this study, the Ramsy model has been extended with CES production function. A model was calibrated for the Iranian economy after solving the optimal control problem and theoretically calculating the long-term growth in the steady state. The theoretical and empirical results of the research are presented as follows.

1- The results indicate that savings rate will be constant and exogenous ($s = s_0$) assuming the Cobb Douglas production function in the growth model. On the other hand, if constant elasticity of substitution production function (CES) is

assumed, then the savings rate is computable from $s_t = \frac{s_0}{e^{\gamma(1-\sigma)t}}$ and has endogenous amount which is estimated according to the model parameters. In saving rate equation s_t, s_0, γ, σ represent saving rate in time t, saving rate in initial time, growth rate of technology, and elasticity of substitution between labor and capital, respectively. So if CES production function is used instead of the Cobb Douglas, the savings rate will be variable over time. On the other hand, saving rate depends on the rate of technology growth. This result can be further investigated in the future studies.

2- The steady economic growth rate is calculable from $g_y = (1-\sigma)\gamma + \left(\frac{\rho - n}{\alpha}\right)^{\sigma} (A_0)^{1-\sigma} - \delta$. In this equation $g_y, \sigma, \gamma, \rho, n, \alpha, A_0, \delta$ denote economics' growth rate, elasticity of substitution between labor and capital growth rate of technology, discount rate, population growth, distribution parameter, initial technology level, and depreciation rate, respectively. If the Cobb Douglas production function (production function with elasticity of substitution equal to one) is used, the growth rate in the steady state will be calculated by ($\frac{\rho - n}{\alpha} - \delta$). With regard to this equation, the growth rate of technology in the long run is not important and it will have no effect on the steady state growth rate. Therefore, the economy steady state growth rate will also depend on the growth rate of technology supposing the elasticity of substitution is less than one. On the other hand, this relationship implies that if we want the growth of technology to have a positive impact on the long-term growth, elasticity of substitution must be less than one. The impact of technology growth on the long-term economic growth will be negative if the elasticity of substitution is greater than one.

3- The elasticity of substitution between capital and labor for the Iranian economy has been 0.45. This indicates that a one-percent increase in the labor force can save 0.45 percent of capital. On the contrary, with a one percent reduction in capital, the workforce must increase by 0.45 percent so that the level of production does not change. Also in the present study, the technology growth rate was estimated to be equal to 0.56 for the Iranian economy.

4- Considering the calculations made for trajectory of capital in the steady state and comparing it with the amount of realized capital, capital was almost on a steady state trajectory and is close to the long-term capital trajectory from 1955 to 2008. The capital was initially out of steady state from 2009 to 2012, but again in 2012 it returned to the steady state. Realized capital was always far from the steady state trajectory from 2013 to 2018. In 2018, we see a 50% gap between realized capital and steady state trajectory. This is due to the uncertainties that have arisen from 2013 to 2018.

5- A comparison of the steady-state trajectory and realized production figures shows that we have always seen economic improvement from 1998 to 2007 until economic prosperity was achieved in 2007. But from 2008 to 2018, the economic downturn was the dominant phenomenon in a way that economic prosperity in 2008 and economic recession in 2018 was witnessed for the Iranian economy. Simultaneous examination of the capital and production trajectories reveals that this recession can be mainly attributed to the reduced capital. This reduction in capital will not be offset unless certainty is restored.

6- Considering the estimated parameters for the Iranian economy, economic growth rate at the steady state and elasticity of substitution of capital and labor have a reverse relationship. Also, the economic growth rate for the Iranian economy in the steady state (the rate of growth that returns the production from the current state to its long-run steady state trajectory) was estimated to be 6.9 percent. On the other hand, if Cobb Douglas production function is assumed (the production function with elasticity of substitution of labor and capital equal to one), the steady state growth rate will approach zero. This conclusion does not seem to be correct. Thus, the results will be closer to the reality using CES production function instead of Cobb Douglas in an endogenous growth model.

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